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Technical Notes

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Radiation Effects on Magnetohydrodynamic Free Convection Flow

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Nomenclature

a_n	=	coefficients in the phase function
f	=	dimensionless stream function, $(\nu u_\infty x)^{-(\frac{1}{2})}\varphi(x,\eta)$
g	=	dimensionless function, $(1/H_0)(\nu x/u_\infty)^{-(\frac{1}{2})}\phi(x,\eta)$
H	=	magnetic field
H_0	=	constant magnetic field
h	=	heat transfer coefficient
I	=	dimensionless radiation intensity, $i/4n^2\bar{\sigma}T_\infty^4$
i	=	intensity of radiation
k	=	thermal conductivity
N	=	conduction-radiation parameter, $k\beta_0/4n^2\bar{\sigma}T_\infty^3$
Nu_x	=	local Nusselt number, hx/k
Pr	=	Prandtl number, $\rho\nu c_p/k$
$P_n(\mu)$	=	Legendre polynomial of the first kind of degree n
$p(\mu,\mu')$	=	phase function
Re_x	=	local Reynolds number, $u_\infty x/\nu$
Q^r	=	dimensionless radiative heat flux, $q^r/4n^2\bar{\sigma}T_\infty^4$
q^r	=	radiative heat flux
T	=	temperature
x, y	=	physical coordinates along and normal to the wall
α	=	thermal diffusivity
α_1	=	magnetic diffusivity
β	=	magnetic force number, $\mu_0 H_0^2/\rho u_\infty^2$
β_0	=	extinction coefficient
ε	=	emissivity of plate surface
ζ	=	ratio of the wall to freestream velocity
η	=	nonsimilarity variable, $y(u_\infty/\nu x)^{\frac{1}{2}}$
θ	=	dimensionless temperature, T/T_∞
λ	=	reciprocal of the magnetic Prandtl number, α_1/ν
μ	=	direction cosine
μ_0	=	magnetic permeability
ν	=	kinematic viscosity
ξ	=	nonsimilarity variable, $\beta_0 x Re_x^{-(\frac{1}{2})}$
ρ	=	fluid density
ρ_d	=	diffuse reflectivity of plate surface
$\bar{\sigma}$	=	Stefan–Boltzmann constant
τ	=	optical variable, $\xi\eta$
φ	=	stream function
ω	=	scattering albedo

Subscripts

w	=	wall
x, y	=	along and normal to the wall
∞	=	external flow

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Introduction

THE study of heat transfer of the magnetohydrodynamic flows has received considerable interest, because of it wide applicability in magnetohydrodynamic electrical power generation, geophysics, etc. The forced and free convection boundary layer flow of an electrically conducting fluid in the presence of a strong magnetic field has been studied by Singh and Cowling [1], Sparrow and Cess [2], Riley [3], Kuiken [4], Soundalgekar and Ramana Murty [5], Soundalgekar and Takhar [6], and Hossain and Ahmed [7]. Singh [8] and Chowdhury and Islam [9] have investigated the effect of a magnetic field of an electrically conducting viscoelastic fluid past an infinite plate. Vayjravelu [10,11] studied the heat transfer in an electrically conducting fluid near a stretching surface. Takhar [12], Raptis [13], and Seddeek [14] have investigated the radiation effect with the optically thin limit on magnetohydrodynamic flow.

The purpose of this Note is to investigate the effects of anisotropic scattering on the magnetohydrodynamic free convection flow of an anisotropic scattering fluid over a semi-infinite plate with a diffuse reflecting surface and moved with a constant velocity.

Analysis

Let us consider the steady two-dimensional flow of an electrically conducting fluid past a semi-infinite plate. The x axis is along the plate to the direction of the flow with the origin at the leading edge of the plate and the y axis is taken normal to it. Figure 1 illustrates the physical geometry. The external flow consists of a uniform freestream with velocity u_∞ and temperature T_∞ ($T_w > T_\infty$). A constant magnetic field H_0 is applied parallel to the plate outside the boundary layer. We assume that the normal component of the induced magnetic field H_y vanishes at the wall and the parallel component H_x approaches the given value H_0 at the edge of the boundary layer. The flow is governed by the following boundary layer equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1a}$$

$$\frac{\partial H_x}{\partial x}+\frac{\partial H_y}{\partial y}=0 \tag{1b}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+\frac{\mu_0}{\rho}\left[H_x\frac{\partial H_x}{\partial y}+H_y\frac{\partial H_x}{\partial y}\right] \tag{1c}$$

$$u\frac{\partial H_x}{\partial x}+v\frac{\partial H_x}{\partial y}-H_x\frac{\partial u}{\partial x}-H_y\frac{\partial u}{\partial y}=\alpha_1\frac{\partial^2 H_x}{\partial y^2} \tag{1d}$$

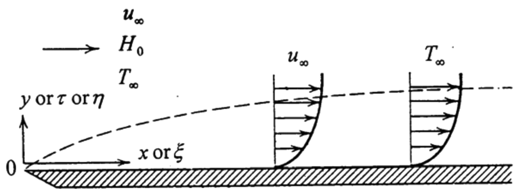

Fig. 1 The geometry and coordinate system of the physical model.

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q^r}{\partial y} \tag{1e}$$

and the boundary conditions are given by

$$u=u_w,\qquad v=H_y=\frac{\partial H_x}{\partial y}=0\qquad T=T_w\quad \text{at } y=0 \tag{2a}$$

$$u\to u_\infty,\qquad H_x\to H_0 T\to T_\infty\quad \text{at } y\to\infty \tag{2b}$$

We introduce the following transformations to recast the boundary layer equations:

$$\theta=\frac{T}{T_\infty},\qquad \theta_w=\frac{T_w}{T_\infty} \tag{3a}$$

$$N=\frac{k\beta_0}{4n^2\bar{\sigma}T_\infty^3},\qquad Q^r=\frac{q^r}{4n^2\bar{\sigma}T_\infty^4} \tag{3b}$$

$$\varphi(x,y)=(\nu u_\infty x)^{(\frac{1}{2})}f(x,\eta) \tag{3c}$$

$$u=\frac{\partial\varphi}{\partial y},\qquad v=-\frac{\partial\varphi}{\partial x} \tag{3d}$$

$$\phi=\left(\frac{\nu x}{u_\infty}\right)^{\frac{1}{2}}H_0 g(x,\eta) \tag{3e}$$

$$H_x=\frac{\partial\phi}{\partial y},\qquad H_y=-\frac{\partial\phi}{\partial x} \tag{3f}$$

$$\eta=y\left(\frac{u_\infty}{\nu x}\right)^{\frac{1}{2}},\qquad \xi=\beta_0 x Re_x^{-(\frac{1}{2})} \tag{3g}$$

$$\beta=\frac{\nu_0 H_0^2}{u_\infty^2},\qquad Pr=\frac{\rho c_p \nu}{k} \tag{3h}$$

$$\lambda=\frac{\alpha_1}{\nu},\qquad \zeta=\frac{u_w}{u_\infty}<1 \tag{3i}$$

These transformations reduce Eq. (1) to

$$f'''+\frac{1}{2}ff''-\frac{1}{2}\beta gg'' =\frac{1}{2}\xi\left(f'\frac{\partial f'}{\partial\xi}-f''\frac{\partial f}{\partial\xi}\right)-\frac{1}{2}\beta\xi\left(g'\frac{\partial g'}{\partial\xi}-g''\frac{\partial g}{\partial\xi}\right) \tag{4a}$$

$$\lambda g'''+\frac{1}{2}fg''-\frac{1}{2}\beta gf'' =\frac{1}{2}\xi\left(f'\frac{\partial g'}{\partial\xi}-g''\frac{\partial f}{\partial\xi}\right)-\frac{1}{2}\xi\left(g'\frac{\partial f'}{\partial\xi}-f''\frac{\partial g}{\partial\xi}\right) \tag{4b}$$

$$\frac{1}{Pr}\theta''+\frac{1}{2}f\theta'=\frac{1}{2}\xi\left(f'\frac{\partial\theta}{\partial\xi}-\frac{\partial f}{\partial\xi}\theta'\right)+\frac{\xi}{NPr}Q^{r'} \tag{4c}$$

The boundary conditions (2) become

$$f=0,\qquad f'=\zeta,\qquad g=g''=0,\quad \text{and}\quad \theta=\theta_w\quad \text{at } \eta=0 \tag{5a}$$

$$f'=1,\qquad g'=0,\quad \text{and}\quad \theta=1\quad \text{at } \eta\to\infty \tag{5b}$$

The radiation part of this problem satisfies the equation of radiative transfer given in the form

$$\mu\frac{\partial I}{\partial\tau}+I=(1-\omega)\frac{\theta^4}{4\pi}+\frac{\omega}{2}\int_{-1}^{1}p(\mu,\mu')I(\tau,\mu')\,d\mu' \tag{6a}$$

where

$$p(\mu,\mu')=\sum_{n=0}^{N}a_nP_n(\mu)P_n(\mu'),\qquad a_0=1 \tag{6b}$$

and the boundary conditions

$$\tau=0,\qquad I^+(0,\mu)=\frac{\varepsilon\theta_w^4}{4\pi}+2\rho_d\int_0^1 I^-(0,-\mu')\mu'\,d\mu',\qquad \mu>0 \tag{7a}$$

$$\tau\to\infty,\qquad I^-(\tau,-\mu)=\frac{1}{4\pi},\qquad \mu>0 \tag{7b}$$

The net heat flux at the wall in terms of the dimensionless quantities is expressed as

$$Q_w=\frac{q_w}{4n^2\bar{\sigma}T_\infty^4}=\left[-N\frac{\partial\theta}{\partial\tau}+Q^r\right]_{\tau=0}=\left[-\frac{N}{\xi}\frac{\partial\theta}{\partial\eta}+Q^r\right]_{\eta=0} \tag{8}$$

The local Nusselt number for the flow becomes

$$Nu_xRe_x^{-(\frac{1}{2})}=\frac{1}{\theta_w-1}\left[-\theta'+\frac{\xi}{N}Q^r\right]_{\eta=0} \tag{9}$$

Results and Discussion

The effects of the ratio of the wall to the freestream velocity ζ on the local Nusselt number are shown in Fig. 2. It is seen that the increase in the value of ζ increases the local Nusselt number.

The influences of the scattering albedo ω and the forward–backward scattering parameters a_1 on the value of the local Nusselt number are represented in Fig. 3. The effect of the scattering albedo is to decrease the value of the total heat flux at small values of ξ, but to increase at large values of ξ and the forward–backward scattering parameter a_1 has a significant effect on the heat transfer. The value of the local Nusselt number for a strong backward scattering fluid ($a_1=-1$) with $\omega=0.1$ is about 2.3% less than that for a strong forward scattering fluid ($a_1=1$) at $\xi=0.12$, and 4.4% less at $\xi=0.6$ and the respective percentages are 26.5 and 47% for $\omega=0.9$. The anisotropic scattering effects on the value of the local Nusselt number are amplified at large values of ω.

Figure 4 represents the variation of the local Nusselt number with the conduction–radiation parameter N, which characterizes the

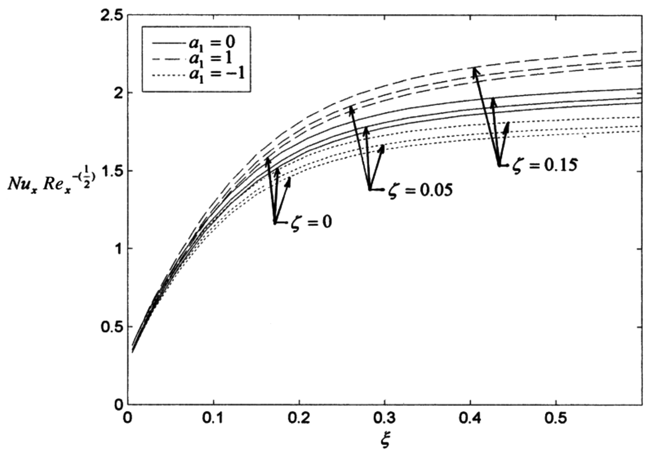

Fig. 2 The effects of the ζ on the value of the Nusselt number for $\beta = 0.5$, $\lambda = 5$, $Pr = 0.733$, $N = 0.1$, $\omega = 0.5$, $\rho_d = 0$, and $\theta_w = 1.2$.

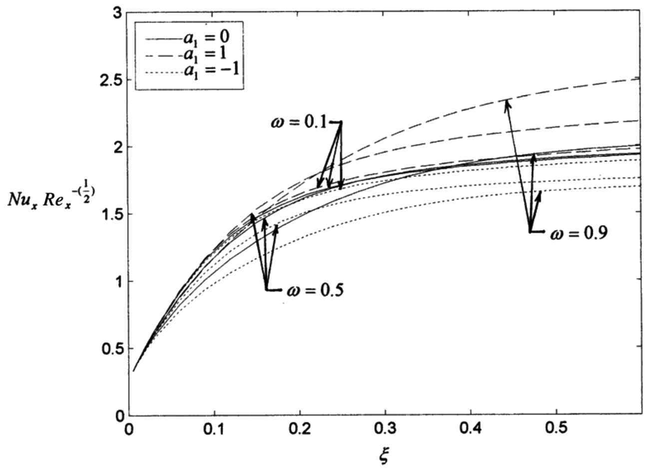

Fig. 3 The effects of the scattering albedo ω and the forward–backward scattering parameter a_1 on the value of the Nusselt number for $\beta = 0.5$, $\lambda = 5$, $Pr = 0.733$, $N = 0.1$, $\rho_d = 0$, $\theta_w = 1.2$, and $\zeta = 0$.

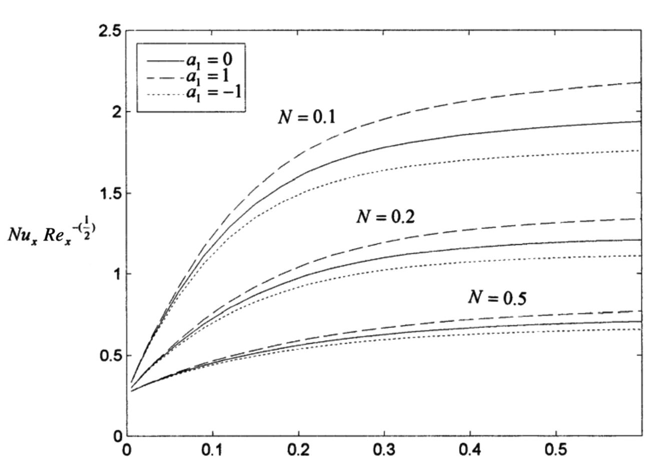

Fig. 4 The effects of the conduction–radiation parameter N on the value of the local Nusselt number for $\beta = 0.5$, $\lambda = 5$, $Pr = 0.733$, $\omega = 0.5$, $\rho_d = 0$, $\zeta = 0$, and $\theta_w = 1.2$.

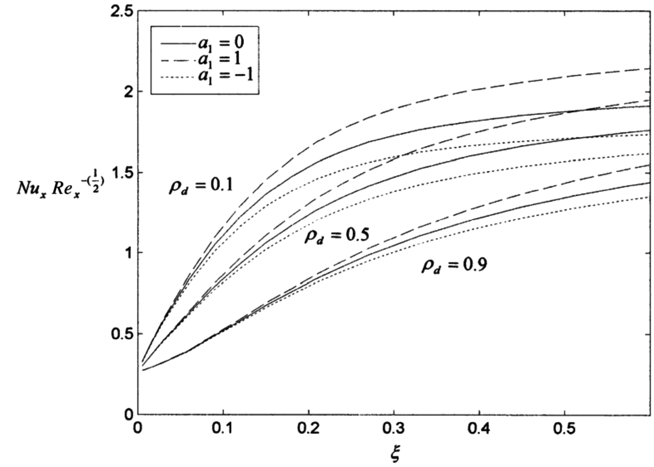

Fig. 5 The effects of the surface reflectivity ρ_d on the value of the local Nusselt number for $\beta = 0.5$, $\lambda = 5$, $Pr = 0.733$, $\omega = 0.5$, $N = 0.1$, $\zeta = 0$, and $\theta_w = 1.2$.

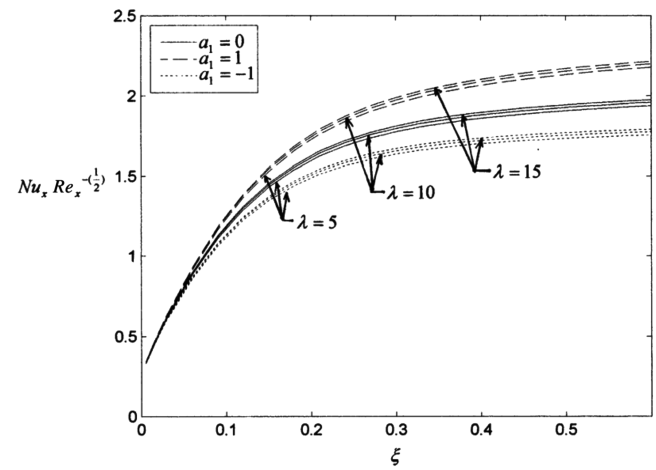

Fig. 6 The effects of the reciprocal of the magnetic Prandtl number λ on the value of the local Nusselt number for $\beta = 0.5$, $Pr = 0.733$, $\omega = 0.5$, $N = 0.1$, $\rho_d = 0$, $\zeta = 0$, and $\theta_w = 1.2$.

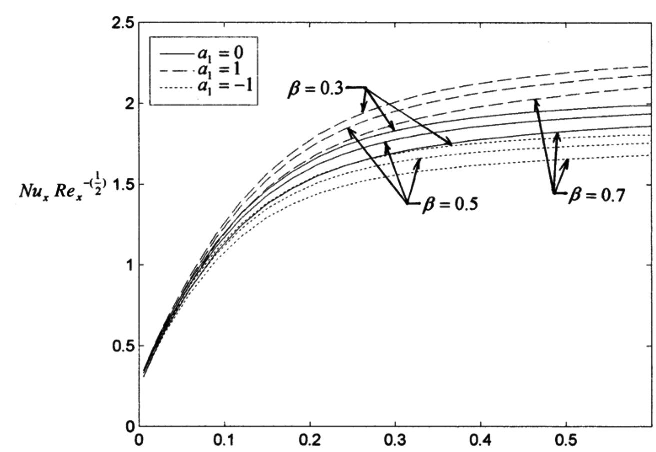

Fig. 7 The effects of the magnetic force number β on the value of the local Nusselt number for $\lambda = 5$, $Pr = 0.733$, $\omega = 0.5$, $N = 0.1$, $\rho_d = 0$, $\zeta = 0$, and $\theta_w = 1.2$.

relative importance of radiation in regard to conduction. It is observed that the value of the local Nusselt number increases as the value of N decreases, and the anisotropic scattering effects on the heat flux are amplified at small values of N.

Figure 5 shows the influence of the surface reflectivity. It is found that the value of the local Nusselt number decreases with the increase of surface reflectivity ρ_d. This is because the reflection at the surface increases the path length of the radiative transfer.

The effects of the reciprocal of the magnetic Prandtl number λ on the value of the local Nusselt number are illustrated in Fig. 6. It is noticed that the value of the local Nusselt number increases as the value of λ increases.

The influences of the magnetic force number β on the value of the local Nusselt number are represented in Fig. 7. It can be seen that the value of the local Nusselt number is reduced as the value of the magnetic force number β increases.

Conclusions

The problem of radiation effects on the magnetohydrodynamic free convection flow of an absorbing, emitting, and anisotropic scattering fluid over a semi-infinite plate has been analyzed. From this study, it is found that

1) The increase in the value of the ratio of the wall to the freestream velocity ζ increases the local Nusselt number.

2) The influence of the optical thickness τ_∞ is that the value of the net radiative heat flux and the local Nusselt number decrease as the optical thickness τ_∞ increases.

3) The effect of the scattering albedo ω is to decrease the value of the local Nusselt number at small values of ξ but to increase at large values of ξ.

4) The forward–backward scattering parameter a_1 has a significant effect on the heat transfer.

5) The increase in the value of the local Nusselt number reduces the value of the conduction–radiation parameter N.

6) The value of the local Nusselt number decreases with the increase of the value of surface reflectivity ρ_d.

7) The value of the local Nusselt number increases as the value of the reciprocal of the magnetic Prandtl number λ increases.

8) The value of the local Nusselt number reduces as the value of the magnetic force number β increases.

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